Legislative Bargaining under Weighted Voting

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Organizations often distribute resources through weighted voting. We analyze this setting using a noncooperative bargaining game based on the Baron-Ferejohn (1989) model. Unlike analyses derived from cooperative game theory, we find that each voter's expected payoff is proportional to her voting weight. An exception occurs when many high-weight voters exist, as low-weight voters may expect disproportionately high payoffs due to proposal power. The model also predicts that, ex post, the coalition formateur (the party chosen to form a coalition) will receive a disproportionately high payoff. Using data from coalition governments from 1946 to 2001, we find strong evidence of such formateur effects. (*JEL* D7, D72)

Collective decision-making frequently involves situations in which actors have different numbers of votes. Some institutions assign unequal voting weights explicitly. Examples include important political bodies, such as the International Monetary Fund, the European Union (E.U.) Council of Ministers, and the U.S. Electoral College, as well as many economic cartels, such as the International Coffee Council. Choice of weights is a subject of ongoing controversy in such bodies. A wider class of problems involving weighted voting arises when blocs of votes are assembled and cast together. Important examples are the formation of coalition governments, voting in legislatures with unified factions, and shareholder voting in corporations.

How does the distribution of votes affect who gets what? Most theoretical and applied analyses of weighted voting employ power indices such as the Shapley-Shubik value, the Banzhaf index, and the Deegan-Packel index.[1] These indices often produce highly nonlinear relations between expected outcomes and votes. Typically, players with the largest weights receive disproportionately high expected payoffs. Many theorists see these nonlinearities as natural reflections of the subtleties of power. For example, William F. Lucas (1978, p. 184) writes: "It is fallacious to expect that one's voting power is directly proportional to the number of votes he can deliver. ... Power is not a trivial function of one's strength as measured by his number of votes. Simple additive or division arguments are not sufficient, but more complicated relations are necessary to understand the real distribution

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[1] See Lloyd S. Shapley (1953), Shapley and Martin Shubik (1954), John F. Banzhaf, III (1965), and John Deegan, Jr. and Edward W. Packel (1978) for definitions of these indices. Applications include Banzhaf (1968), Guillermo Owen (1975), Samuel Merrill III (1978), Jacob S. Dreyer and Andrew Schotter (1980), Manfred J. Holler (1982), Robert H. Bates and Da-Hsiang Donald Lien (1985), Amnon Rapoport and Esther Golan (1985), George Rabinowitz and Stuart Elaine MacDonald (1986), Kaare Strom et al. (1994), Thomas Konig and Thomas Brauninger (1996), Emilio Calvo and J. Javier Lasaga (1997), and Sven Berg et al. (1993). Other cooperative solution concepts applied to weighted voting games include bargaining sets, bargaining aspirations, the kernel, and the competitive solution. See, for example, Norman Schofield (1976, 1978, 1982, 1987), Richard D. McKelvey et al. (1978), Elaine Bennett (1983a, 1983b), and Holler (1987).

of influence."[2] These arguments have had a significant impact on the field of political economy and debates about the design of constitutions. For example, a large amount of literature has emerged employing power indices to study the voting weights in the government of the European Union.[3]

While widely used, power indices and the arguments they justify run contrary to an important intuition. Elementary microeconomic theory teaches that in competitive situations perfect substitutes have the same price.[4] In a political setting in which votes might be traded or transferred in the formation of coalitions, one might expect the same logic to apply. If a player has k votes, then that player should command a price for those votes equal to the total price of k players that each have one vote. In terms of expected payoffs, the player with k votes should expect to have a payoff k times as great as the payoff expected by a player with one vote. If "expected payoff" can be used as a measure of "power," then the player with k votes should also expect to have k times as much power as the player with one vote.[5]

In this paper, we present a straightforward model of divide-the-dollar politics which captures the simple price-theoretic intuition. We show that the noncooperative bargaining model of David P. Baron and John A. Ferejohn (1989) leads naturally to the result that expected payoffs are proportional to voting weights.[6] While cooperative game theory concepts apply only to voting power, this model captures both voting power and proposal power.[7] In the Baron-Ferejohn model, a randomly drawn legislator makes a proposal—a division of the dollar—which is then put to a vote. Proposers seek to offer as little of the dollar as possible to others, because they keep the residual for themselves.

As in most noncooperative bargaining models, we must make an assumption about the probability that each legislator is chosen to make a proposal. We analyze two natural cases. First, legislators' proposal probabilities may be proportional to their voting weights. This is consistent with the empirical pattern found in the formation of coalition governments.[8] Second, legislators' proposal probabilities may all

[2] Another example is Steven J. Brams and Paul J. Affuso (1985, p. 138): "A measure like Banzhaf's is not only an eminently reasonable indicator of a crucial aspect of voting power—the ability of a member to change an outcome by changing its vote—but also highlights the fact that size (as reflected by voting weights) and voting power may bear little relationship to each other."

[3] See Brams and Affuso (1985), Berg et al. (1993), Kaisa Herne and Hannu Nurmi (1993), Madeline O. Hosli (1993, 1995). Mika Widgren (1994, 2000), R. J. Johnston (1995), Jan-Erik Lane and Reinert Maeland (1995, 1996), Lane et al. (1995, 1996), Matthias Bruckner and Torsten Peters (1996), Anthony L. Teasdale (1996), George Tsebelis and Geoffrey Garrett (1996), Ulrich Bindseil and Cordula Hantke (1997), Dan S. Felsenthal and Moshe Machover (1997, 2000, 2001), Annick Laruelle and Widgren (1998), Konig and Brauninger (1998), Garrett and Tsebelis (1999a, 1999b, 2001), Holler and Widgren (1999), Nurmi and Tommi Meskanen (1999), and Matthias Sutter (2000a, 2000b).

[4] Power indices are criticized on other grounds as well. Some critics emphasize the paradoxical flavor of the predictions that power indices yield (Brams and Affuso, 1985; Holler, 1987; Felsenthal and Machover, 1998; Garrett and Tsebelis, 1999a, 1999b, 2001). A further well-known problem is that these indices capture only the influence that comes from having votes, and ignore important sources of power, especially the ability to make proposals (James G. March, 1955). Moreover, these indices typically do not have a basis in non-cooperative game theory; an exception is Faruk Gul (1989), who provides a non-cooperative justification of the Shapley value.

[5] Theorists working on this problem commonly equate power and expected payoffs. There is some debate over whether the definition of power should also include the ability to change the outcome, even though the action does not result in an increase in, and may even lower, the payoff for the pivotal actor, e.g., William H. Riker (1964, p. 344). In this paper we use the terms power and expected payoffs interchangeably.

[6] The Baron-Ferejohn model extends the Ariel Rubinstein (1982) bargaining model to the case of three or more players operating under simple majority rule. This is the most widely used model of legislative bargaining, and has been used extensively to study various aspects of distributive politics and government institutions. See Joseph Harrington (1989, 1990a, 1990b), Baron (1991, 1996, 1998), McKelvey and Raymond Riezman (1992), Baron and Ehud Kalai (1993), Randall Calvert and Nathan Dietz (1996), Eyal Winter (1996), V. V. Chari et al. (1997), Daniel Diermeier and Timothy J. Feddersen (1998), Jeffrey S. Banks and John Duggan (2000), William LeBlanc et al. (2000), Nolan M. McCarty (2000a, 2000b), Hulya Eraslan (2002), Peter Norman (2002), and Ansolabehere et al. (2003). Other non-cooperative models of n-person bargaining include Reinhardt Selten (1981), Kenneth Binmore (1987), Gul (1989), Kalyan Chatterjee et al. (1993), and Benny Moldovanu and Winter (1995), Sergiu Hart and Andreu Mas-Colell (1996), and Akira Okada (1996). To our knowledge, none of these has been applied to the study of legislative politics.

[7] Massimo Morelli (1999) develops a model of "demand bargaining" that also captures these features and arrives at proportional payoffs, albeit in a different bargaining environment. We discuss his model and results further below.

[8] Diermeier and Antonio Merlo (2004) estimate that parties' "recognition probabilities" depend strongly on their voting weights, as well as which party formed the previous government.

be equal. This is the most common assumption in the theoretical literature. It is also consistent with an institution such as the E.U.'s Council of Ministers, in which countries have unequal voting weights but—at least theoretically—equal opportunities for making proposals.

In the first case, we find that each player's expected payoff (continuation value) is equal to his share of the total voting weight. The intuition underlying this result is straightforward and follows from a simple substitution argument. Suppose type-1 players have a continuation value of 2 and a voting weight of 1, while type-2 players have a continuation value of 5 and a voting weight of 2. Then rational proposers seeking to minimize the costs of the coalitions they construct will never include type-2 players in their coalitions (except, perhaps, because of "integer" issues). Proposers will substitute type-1 players for type-2 players whenever possible, since two type-1 players have the same total voting weight as one type-2 player, but a total cost that is four-fifths as much.

In the second case there are two types of equilibria, depending on the distribution of voting weights. The first is an "interior" equilibrium in which each player's expected payoff is equal to her share of the total voting weight—just as in the case with proportional recognition probabilities. This occurs when the distribution of voting weights is not too skewed.

The second type is a "corner" equilibrium. In this case the players can be divided into two distinct groups, defined by some cutoff weight t_0. Players with voting weights less than t_0 have expected payoffs that are greater than their shares of the total voting weights, while players with voting weights greater than (or equal to) t_0 have expected payoffs that are less than their shares of the total voting weight. All players with voting weights greater than (or equal to) t_0 have expected payoffs that are equal to some θ times their voting weight, with $\theta < 1$. The reason weak players have expected payoffs that are greater than their shares of the voting weight is their proposal power. By assumption, this is assigned equally to all players. The corner equilibria occur when the weakest players are so weak that, even if no other proposers ever include them as coalition partners, their proposal power alone is enough to yield an expected payoff greater than their share of the voting weight.

The logic in the noncooperative model differs sharply from that of the power indices. Power indices are based on the idea that all orderings (or winning coalitions, or minimal-winning coalitions) are *equally likely* to form, regardless of how expensive or cheap they are. Under competitive bargaining, expensive coalitions will form rarely or not at all, and cheap coalitions will form quite often.

The competitive bargaining logic produces several key predictions that differ from power indices. First, as noted, at an interior equilibrium the expected payoffs of players depend linearly on the number of votes. Second, in the corner equilibria, it is the *weaker* players who receive expected payoffs greater than their weight. By contrast, power indices typically assign disproportionate power to players with higher voting weights. Third, even "dummy" players can have positive expected payoffs, because all players have some chance of being chosen as proposer. By contrast, power indices assign zero power to any player with too few votes to be pivotal in at least one coalition.

The results also suggest that the linear payoffs of the interior equilibrium are closely related to low proposal power. Supermajoritarian requirements reduce proposal power and help to produce the interior equilibrium. Setting recognition probabilities to be proportional to voting weights also restores linearity in the payoffs. We therefore conjecture that institutional features such as an open rule will similarly make linear payoffs more likely.

In addition to the theoretical analysis, we test predictions of the model using data on coalition governments in parliaments from 1946 to 2001. An extensive literature examines the empirical relationship between parties' shares of parliamentary seats and their shares of cabinet posts in coalition governments to study theoretical conjectures about bargaining generally. A critical weakness of this literature is that it focuses on the effects of *seat shares* on shares of cabinet posts, but the theoretically relevant concept is *shares of voting weights.* We correct this problem and offer the first empirical study of the relationship between parties' shares of voting weights and their shares of cabinet government posts. Consistent with the Baron-Ferejohn framework, we find that the party recognized to form the government receives a share of posts much larger than its voting weight.

Our theoretical results build on an important literature that uses noncooperative game theory to study distributional politics under varying institutional settings, such as differential voting weights, unicameral and bicameral legislatures, and supermajority rules. Baron and Ferejohn (1989) develop the basic closed rule bargaining model in a unicameral legislature when all players have equal voting weight. Banks and Duggan (2000) provide an existence result, but not a characterization, for a generalized Baron-Ferejohn model that encompasses weighted voting. A series of papers characterizes the outcomes of the Baron-Ferejohn model under different institutional arrangements, including veto-players and unanimity rules. Winter (1996) and McCarty (2000a) study variants of the Baron-Ferejohn model with veto players, and Chari et al. (1997) and McCarty (2000b) incorporate an executive veto. Merlo and Charles Wilson (1995) study the equilibria of a related game under unanimity rule. Baron (1998) and Diermeier and Feddersen (1998) compare legislatures with and without a "vote of confidence" procedure. McKelvey and Riezman (1992) analyze the effects of seniority rule. Ansolabehere et al. (2003) characterize the equilibria in a bicameral setting (also see Diermeier and Myerson, 1999). LeBlanc et al. (2000) show how different institutional arrangements affect the level of public investments. Baron (1996), Calvert and Dietz (1996), Diermeier and Merlo (2000), and Matthew Jackson and Boaz Moselle (2002) analyze variants of the Baron-Ferejohn model with externalities and/or spatial preferences. To our knowledge, no previous analysis provides a characterization of the generalized model under weighted voting with majority or supermajority voting. That is the contribution of the current paper.

I. Model and Results

A. *The Model*

Our model is based on the closed-rule, divide-the-dollar game studied by Baron and Ferejohn (1989). Players in the game belong to a set $\mathbf{N}$ of legislators, where $|\mathbf{N}| = n \geq 3$. Legislators have preferences over the funds allocated to their district, with each legislator $i \in \{1, 2, \ldots, n\}$ receiving utility $u_i(\mathbf{x}) = x_i$ from a division of the dollar $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in X$, where $X \equiv \{\mathbf{x} | x_i \geq 0 \ \forall i, \sum_{i=1}^{n} x_i \leq 1\}$. Each legislator is of an integral *type* $t \in \{1, 2, \ldots, T\}$, where each type has a unique voting weight and $1 \leq T \leq n$. Let $\mathbf{T}$ denote the set of types, and let $t(i)$ denote the type of legislator i. The number of type-t legislators is n_t, where n_t is positive and integer-valued, so $\sum_{t=1}^{T} n_t = n$. We denote a type-t legislator's voting weight w_t, and assume that: (a) weights are positive integers, and (b) $w_t < w_{t+1}$ for all $t < T$. For convenience, we arrange the legislators so that legislators $n_{t-1} + 1, \ldots, n_{t-1} + n_t$ each have weight w_t, where $n_0 = 0$. Thus, legislators are numbered in increasing order of weight.

The legislature uses a generalized majority rule. For any coalition of legislators $\mathbf{C} \subseteq \mathbf{N}$, let $n_t(\mathbf{C})$ represent the number of type t legislators contained within. Let $w(\mathbf{C}) = \sum_{t=1}^{T} n_t(\mathbf{C}) w_t$ represent its total voting weight, and let $w = w(\mathbf{N}) = \sum_{t=1}^{T} n_t w_t$ be the combined weight of all legislators. A coalition $\mathbf{C}$ is winning if and only if $w(\mathbf{C}) > \underline{w}$, where $\underline{w}(w - w_T > \underline{w} > w/2)$ is the proportion of the total voting weight required for passage. Let $\mathcal{W}$ denote the set of winning coalitions.

Weighted voting introduces a significant technical complication to this framework. A weighted voting game is *homogeneous* if all minimal winning coalitions have exactly the same total voting weight. All (strong) games with up to five players are homogeneous. For larger populations of players, however, games are sometimes nonhomogeneous, and rounding problems may arise. For example, in a six-member majority-rule legislature, all minimum winning coalitions are of size or "weight" 4 under unweighted voting. But if votes are weighted (9, 5, 5, 3, 2, 2) and 15 of 26 votes are required for victory, then some minimum winning coalitions have a total weight of 15 (coalitions consisting of players with weights 5, 5, 3, and 2), while others have 16 or more. Furthermore, the weight-9 player cannot be in a minimum-weight, minimum-winning coalition, since any winning coalition that includes her has a weight of at least 16.

Nonhomogeneity affects the general characterization of proposal strategies. As the example illustrates, some types of proposers may find it necessary or optimal to choose coalitions that are not of the smallest winning voting weight.

We address this problem by examining the behavior of "replicated" voting games. That is, we examine equilibrium strategies as the number of players of each type is multiplied by some positive integer, $r \in \mathbb{Z}_+$. The basic game described above has $r = 1$, and a game with r replications has rn players, a total weight of rw, and a threshold for victory of $r\underline{w}$. We show that the effect of nonhomogeneity becomes small as r increases, thus allowing us to derive some general results. We also show, however, that these results hold at $r = 1$ for homogeneous games, and that for most "small" games the effect of nonhomogeneity is small even when $r = 1$.

The sequence of the game is as follows. In each period, Nature randomly recognizes a proposer from **N**. Recognition probabilities are equal among legislators of a given type, and draws are i.i.d. across periods. Thus, in each period a type-t legislator's recognition probability is p_t. As noted in the introduction, we will examine two cases: one in which recognition probabilities are uniform and one in which they are proportional to voting weights. The proposer proposes a division of the dollar from X. All legislators then vote for or against the proposal. If the proposal receives weight $r\underline{w}$ in support, then the dollar is divided and the game ends. If the proposal is rejected, then a new proposer is randomly drawn and the game continues. We study the infinite-horizon game, with no discounting. The game can be treated as a sequence of identical subgames, where each subgame begins with Nature's move to draw a proposer.

The solution concept used here is stationary equilibrium. Strategies under this concept are stationary; thus, each player uses history-independent strategies at all proposal-making stages, and voting strategies depend only on the current proposal. We therefore omit reference to time periods and game histories to conserve on notation. A proposal strategy for legislator i is then $\chi_i \in \Delta(X)$, where $\Delta(X)$ is the set of probability distributions over X. A voting strategy for legislator i is simply a mapping from the offered amount to a vote $\phi_i : X \rightarrow \{0, 1\}$, where 1 is a vote in favor.

In a stationary equilibrium, strategies must satisfy sequential rationality and weak dominance. Stationary equilibria therefore have the following properties. Each player has a continuation value which gives her expected payoff prior to Nature's draw of a proposer. By stationarity, this value is time-invariant. A player will vote for a proposal if it is at least her continuation value, and against it otherwise. Finally, for any legislator i recognized as proposer, if the set of proposals $\mathcal{X}$ that provides her with at least her continuation value and will be accepted as non-empty, then i must propose an element of $\arg\max_{\mathbf{x} \in \mathcal{X}} u_i(\mathbf{x})$.[9]

B. *Results*

In this section, we prove the existence of and characterize a stationary equilibrium to the legislative bargaining with weighted voting. Specifically, there exists a stationary equilibrium in which the first legislator recognized makes a proposal that wins approval and ends the game. These equilibria are symmetric, in the sense that any two players with the same voting weight have the same continuation value. And, for sufficiently large games, the legislators' relative prices are proportionate to their voting weights.

Existence follows by establishing our model as a special case of Banks and Duggan (2000).

PROPOSITION 1: *A stationary no-delay equilibrium exists.*

PROOF:

All proofs are in the Appendix.

An important feature of these stationary equilibria is their symmetry. Types of legislators are defined by their voting weight. Any two legislators of the same type bring the same number of votes to any potential coalition, and they have the same probability of being chosen to form a coalition in the future. As a result, stationary equilibria are "symmetric" in the following sense.

COMMENT 1: *In a stationary equilibrium, the continuation values for players of the same type are equal.*

The result implies that, at the beginning of each subgame, players of each type t will have the same continuation value, v_t. This property

[9] If $\mathcal{X}$ is empty, then i proposes an allocation that will be rejected, and receives her continuation value in expectation. This possibility does not arise in the games we study.

allows us to narrow the set of proposals that may occur in equilibrium. At a stationary equilibrium, the proposer must offer at least v_t to a type-t player in order to obtain that player's support. Since proposers wish to minimize their offers, every legislator must be offered either v_t or 0 in equilibrium. For each coalition $\mathbf{C}$, let $v(\mathbf{C}) = \sum_{t=1}^{T} v_t n_t(\mathbf{C})$ be the total "cost" of $\mathbf{C}$. For a proposing legislator i of type t, let $\underline{v}_t = \min_{\{\mathbf{C}|i \subseteq \mathbf{C}, \mathbf{C} \in \mathcal{W}\}} v(\mathbf{C} \backslash \{i\})$ be the minimum total payment proposed to coalition partners besides herself. The proposer retains the surplus, and thus receives $1 - \underline{v}_t$.

The result also implies that in equilibrium, the ex ante expected payoffs will be identical among legislators of a given type. The ex post distribution will heavily favor the proposer, however, because the proposer is the residual claimant. Since most empirical research examines data that are the outcome of the coalition process (rather than the expected outcome), the ex post distribution will be of considerable use in testing this model. The ex ante share is what one would expect across a large number of replications of a given situation, and it is of considerable importance in thinking about the fairness of any given distribution of votes and voting rule.

What share of the overall value can coalition partners expect to receive in the stationary equilibria? A key intuition behind our results is the idea of a legislator's relative "price." The price a type-t coalition partner can command equals that player's continuation value, v_t, divided by his or her share of the voting weight in the rth replication, i.e., $w_t/(rw)$. That is, the legislator's relative price is $\theta_t = v_t rw/w_t$ for a type-t legislator. If a legislator is relatively expensive to include in a coalition, then the continuation value for that legislator is high relative to the share of votes they would bring to a coalition, implying a price of $\theta_t > 1$.

Our results require a general expression for v_t. Let q_t be the probability that a type-t legislator is chosen as a coalition partner, given that she is not the proposer. Stationarity implies that

$$v_t = p_t(1 - \underline{v}_t) + (1 - p_t)q_t v_t. \tag{1}$$

Rearranging, we can write a type-t legislator's continuation value as

$$v_t = \frac{p_t(1 - \underline{v}_t)}{1 - (1 - p_t)q_t}. \tag{2}$$

The exact characterization of the continuation value depends on the nature of the recognition probabilities. We begin by considering the case in which recognition probabilities are proportional to voting weight; i.e., $p_t = w_t/(rw)$.

PROPOSITION 2: *Suppose $p_t = w_t/(rw)$ for all t. There exists a finite $\bar{r}_1$ such that if $r > \bar{r}_1$, then in any stationary equilibrium $v_t = w_t/(rw)$ (i.e., $\theta_t = 1$) for all t.*

The proof of Proposition 2 works by establishing the impossibility of a legislator type for which $\theta_t < 1$. This is because on average, any such cheap legislator must be chosen as a coalition partner with probability too high to keep her expected payoff below her share of the voting weight, $w_t/(rw)$.

Two features of the equilibria identified in Proposition 2 are worth noting. First, the equilibrium payoffs are unique, and thus the stationary equilibrium is unique up to differences in the probabilities of choosing coalition partners that do not affect the players' expected payoffs. Second, in equilibrium, for all t we can solve for $\underline{v}_t$ and q_t.

$$\underline{v}_t \approx \underline{w}/w - v_t = (r\underline{w} - w_t)/(rw) \tag{3}$$

$$q_t \approx \frac{r\underline{w} - w_t}{rw - w_t}. \tag{4}$$

Note that each type's probability of being included in a coalition is slightly decreasing in w_t, but the probabilities are all approximately equal for large r.

We now turn to the second case, where recognition probabilities are constant across players; i.e., $p_t = 1/(rn)$. This case corresponds most closely to the original model of Baron and Ferejohn (1989), and is also useful for illustrating the effects of different distributions of proposal power.

There are two subcases, which depend on the distribution of voting weight across types of legislators. In the first, an "interior" equilibrium arises and prices are linear in weights. In the second, "cornering" is possible and some prices are non-linear. In both, a type-t

player's continuation value is given by substituting p_t into (2):

$$v_t = \frac{1 - \underline{v}_t}{rn - (rn - 1)q_t}. \tag{5}$$

The following proposition characterizes the stationary equilibrium for the interior subcase. Recall that $w_1 = \min_t\{w_t\}$. In any stationary equilibrium the expected payoff (continuation value) of each legislator is proportional to his voting weight at an interior solution.

PROPOSITION 3: *Suppose $p_t = 1/(rn)$ for all t. Also, suppose $n > (w - \underline{w})/w_1$. There exists a finite $\bar{r}_2$ such that if $r > \bar{r}_2$, then in any stationary equilibrium $v_t = w_t/(rw)$ (i.e., $\theta_t = 1$) for all t.*

As with Proposition 2, this distribution of payoffs is unique. Note also that in the case of simple majority rule ($\underline{w} \approx w/2$), the condition in Proposition 3 becomes $n > w/(2w_1)$. This implies that proportional payoffs will occur when there are many low-weight players in the population.

We may compare the result here with Proposition 2 by solving (2) to obtain an approximate expression for q_t:

$$q_t \approx 1 - \frac{r(w - \underline{w})}{(rn - 1)w_t}. \tag{6}$$

Clearly, q_t is increasing in w_t. Thus, in contrast with (4), one of the main reasons that types with higher voting weights receive higher expected payoffs in equilibrium is that proposers are more likely to choose them as coalition partners.

The next proposition characterizes the stationary equilibrium in the subcase where the conditions of Proposition 3 are not satisfied. In this subcase, linear payoffs cannot be sustained in equilibrium, and the equilibrium payoffs are at a "corner." Types with the *smallest* voting weights have expected payoffs greater than their relative weight, while those with the largest weights receive expected payoffs lower than their relative weight. Additionally, those with relatively large voting weight have expected payoffs that are proportional to voting weights, although it is unclear whether proportionality holds for those with relatively small voting weight.

PROPOSITION 4: *Suppose $p_t = 1/rn$ for all t. Also, suppose $n \le (w - \underline{w})/w_1$. There exists a finite $\bar{r}_3$ such that if $r > \bar{r}_3$, then in any stationary equilibrium there is a unique type $t_0 = \max\{t | n \le (w - \underline{w})/w_t\}$ and a number $\theta = \min\{\theta_t\} < 1$ such that $v_t = \theta w_t/(rw)$ for all $t > t_0$ and $v_t > w_t/(rw)$ for all $t < t_0$.*

The proof of Proposition 4 also establishes that if $n < (w - \underline{w})/w_{t_0}$, then type t_0 receives a disproportionately large share. This condition is automatically satisfied if, for example, $(w - \underline{w})/w_{t_0}$ is non-integral. If $n = (w - \underline{w})/w_{t_0}$, however, then type t_0 may belong to the set of types receiving proportional payoffs.

It is worth noting the role played by "cheap" and "expensive" types in equilibrium. Cheap types must constitute at least $\underline{w}/w$ of the total voting weight, for otherwise each cheap legislator would almost always be chosen as a coalition partner. Using (5), this can be shown to be impossible. This implies that coalitions can be constructed using cheap legislators almost exclusively, and thus $q_t \approx 0$ for all expensive types.[10] Expensive legislators are therefore almost never included in winning coalitions.

C. *Discussion*

In the corner equilibrium of Proposition 4, the high voting weight types are "underpaid" relative to their voting weight, while low-weight types are "overpaid." This is the opposite of what tends to happen for many of the power indices.

The intuition behind this result is that equal proposal probabilities disproportionately benefit voters with low weights. A corner equilibrium occurs when the expected payoff to some low-weight voter is greater than his share of the weight, even when no other proposers *ever* choose him as a coalition partner. The high payoff that occurs in the event that he is a proposer determines his entire expected payoff.

[10] The equilibrium payoffs are "approximately" unique, in the sense that the variations from substituting $q_t = 0$ into (5) for types $t \le t_0$ diminish to zero as $r \to \infty$.

This suggests that if recognition probabilities more closely approximated voting weights, then linearity in expected payoffs and voting weights would be restored. When recognition probabilities coincide exactly with voting weights, Proposition 2 predicts payoffs that are proportional to voting weights.

Note also that corner equilibria are more likely to occur when the threshold for victory $\underline{w}$ is low, since lower values of $\underline{w}$ imply greater benefits to being proposer. Thus, we expect linear payoffs when $\underline{w}$ is high, that is, when the collective choice rule is supermajoritarian. An open rule also makes proposing less valuable, since the proposer must offer higher payoffs to his coalition partners and must often build supermajorities to reduce the probability of counter-proposals (see Baron and Ferejohn, 1989). Thus, we also expect that linear payoffs will be more likely under an open rule.

Finally, a comparative static result follows immediately. Except for types that "corner," a small increase in voting weight always increases a player's expected payoff, but a small increase in proposal probability does not. The types that corner have low voting weights. For these types, a small increase in voting weight carries no benefit, since these types will still "never" be included in a coalition. A small increase in proposal probability, however, increases the expected payoff for the types that corner.

II. Examples of Small Legislatures

The propositions above apply to large legislatures. The basic logic underlying these results, however, holds for relatively small legislatures as well.

Before proceeding, we must revisit the matter of voting weights. Each finite-weighted voting game can be represented by many different vectors of actual votes—that is, there are different distributions of votes that produce the exact same set of winning coalitions. Consider two legislatures with 101 seats. In one, party A has 50 seats, party B has 50 seats, and party C has 1 seat. In the other, party A has 34 seats, party B has 34 seats, and party C has 33 seats. These have the same set of minimum winning coalitions: (AB, AC, BC). What vector of weights should we choose to characterize this game?

Isbell (1956) shows that if a game has a homogeneous representation, then this representation is unique, and the voting weights are minimum integer weights. Nonhomogeneous games do not always have a unique representation in minimum integer weights, but most games with small numbers of players do, and even when multiple representations exist the differences between them are usually minor. Thus, in what follows we will use minimum integer weights.[11]

A. *A Comparison of Indices for Small Legislatures*

The Shapley-Shubik and Banzhaf indices represent what players are expected to receive assuming random formation of coalitions. The expected payoffs from the Baron-Ferejohn competitive bargaining model are analogous in that they reflect what players expect to receive averaging over all possible proposers. Here we compare the expected payoffs from competitive bargaining and the Shapley-Shubik and Banzhaf indices for legislators with 3, 4, 5, and 6 members. In all cases we assume all legislators have equal proposal probabilities.

The following example illustrates the general approach to finding the unique equilibrium payoffs (brute force).

Example. Consider the 5-player game with weights (3, 1, 1, 1, 1) and $\underline{w} = 4$ ($w = 7$, so a simple majority of the weight is needed to pass a proposal). This corresponds to row 2 of Table 1. Call the player with a voting weight of 3 legislator A, and call the players with weights of 1 type-B legislators. Let v_A and v_B be the corresponding continuation values. We show that all stationary equilibria yield $v_A = 3v_B$ ($v_A = 3/7$ and $v_B = 1/7$).

Suppose $v_A > 3v_B$. Then legislator A is relatively expensive, and all type-B proposers avoid her whenever possible. When legislator A is the proposer she offers v_B to one type-B legislator

[11] Aaron B. Strauss (2003) offers an algorithm for computing the minimum integer voting weights of any weighted voting game. In all applications examined in this paper, either the derived minimum integer weights were unique, or the differences in the weights were inconsequential.

and keeps $1 - v_B$ for herself. When any type-B legislator is the proposer, that legislator offers v_B to the three other type-B legislators and keeps $1 - 3v_B$ for herself. Thus, v_A and v_B satisfy: $v_A = \frac{1}{5}(1 - v_B)$, and $v_B = \frac{1}{5}(1 - 3v_B) + \frac{1}{5}(\frac{1}{4})v_B + \frac{3}{5}v_B$. (The first term in the expression for v_B is for the event that the given type-B legislator is the proposer. The second term is for the event that legislator A is the proposer. And, the third term is for the event that one of the other three type-B legislators is the proposer.) Solving these two equations yields $v_A = 3/19$ and $v_B = 4/19$. But this contradicts the assumption that $v_A > 3v_B$, so there can be no equilibria with $v_A > 3v_B$.

Next, suppose $v_A < 3v_B$. Then type-B legislators are relatively expensive, and proposers avoid them whenever possible. When legislator A is the proposer she offers v_B to one type-B legislator and keeps $1 - v_B$ for herself. When any type-B legislator is the proposer, that legislator offers v_A to legislator A and keeps $1 - v_A$ for herself. Thus, v_A and v_B satisfy: $v_A = \frac{1}{5}(1 - v_B) + \frac{4}{5}v_A$, and $v_B = \frac{1}{5}(1 - v_A) + \frac{1}{5}(\frac{1}{4})v_B$. (The first term in the expression for v_A is for the event that legislator A is the proposer and the second term is for the event that a type-B legislator is the proposer. The first term in the expression for v_B is for the event that the given type-B legislator is the proposer and the second term is for the event that legislator A is the proposer.) Solving these two equations yields $v_A = 1$ and $v_B = 0$. But this contradicts the assumption that $v_A < 3v_B$, so there can be no equilibria with $v_A < 3v_B$.

The only remaining possibility is that $v_A = 3v_B$, in which case proposers do not favor either type. We can construct an equilibrium in which this occurs as follows: When legislator A is the proposer she offers v_B to 1 type-B legislator and keeps $1 - v_B$ for herself. When any type-B legislator is the proposer, that legislator offers v_A to legislator A with a probability of 3/4, and she offers v_B to the three other type-B legislators with a probability of 1/4; in either case she keeps $1 - v_A = 1 - 3v_B$ for herself. Then v_A and v_B satisfy: $v_A = \frac{1}{5}(1 - v_B) + \frac{4}{5}(\frac{3}{4})v_A$, and $v_B = \frac{1}{5}(1 - v_A) + \frac{1}{5}(\frac{1}{4})v_B + \frac{3}{5}(\frac{1}{4})v_B$. Solving these two equations yields $v_A = 3/7$ and $v_B = 1/7$.

Table 1 compares the Shapley-Shubik index, Banzhaf index, and expected payoffs under the competitive bargaining game, for all strong four-player and five-player weighted voting games, and a large sample of six-player games.[12] The table shows that the results derived in the previous section hold for these smaller games. In all but two cases, the competitive bargaining model yields expected payoffs equal to the players' shares of the voting weights. The exceptions occur in the last two rows of the table, where a cornering equilibrium is reached, and in these cases the difference between the expected payoff and proportionality is quite small.

The right-hand side of the table shows the differences between the power indices and the expected payoffs of the competitive bargaining game. In many cases the differences are slight, but in some cases they are large—see, e.g., the five-player game with weights (3, 1, 1, 1, 1) and the six-player game with weights (4, 1, 1, 1, 1, 1) and (4, 3, 3, 1, 1, 1). In all cases, the expected payoffs of the players with the largest weights are lower than their Shapley-Shubik and Banzhaf indices (except in one case where they are the same). In all but one case, the expected payoffs of the players with the smallest weights are lower than their power indices.

Finally, we should note that the basic logic does not necessarily require that the smallest weight be equal to 1. For example, consider the seven-player game (3, 3, 3, 2, 2, 2, 2). This is a nonhomogeneous game in its minimal integer form. It is straightforward to show that any stationary equilibrium produces expected payoffs of ($\frac{3}{17}$, $\frac{3}{17}$, $\frac{3}{17}$, $\frac{2}{17}$, $\frac{2}{17}$, $\frac{2}{17}$, $\frac{2}{17}$).

B. *An Application: Council of Ministers of the European Union*

A prominent example of the application of cooperative game theory solutions is the design of the government of the European Union. Each member country has a single vote in the E.U. Council of Ministers, but to compensate for differences in country size the votes have different weights. An extensive body of literature analyzes the distribution of power in the Council using cooperative game theory solution

[12] A simple game is strong if the complement of a losing coalition is always winning—so, there are no blocking coalitions.

TABLE 1—POWER INDICES VERSUS EXPECTED PAYOFFS

Voting weights	Power indices and expected payoffs Row 1: Shapley-Shubik Row 2: Banzhaf Row 3: *Expected payoffs*						Power indices relative to expected payoffs					
2, 1, 1, 1	0.500	0.167	0.167	0.167			1.25	0.84	0.84	0.84		
	0.500	0.167	0.167	0.167			1.25	0.84	0.84	0.84		
	0.400	*0.200*	*0.200*	*0.200*								
3, 1, 1, 1, 1	0.600	0.100	0.100	0.100	0.100		1.40	0.70	0.70	0.70	0.70	
	0.636	0.091	0.091	0.091	0.091		1.48	0.64	0.64	0.64	0.64	
	0.429	*0.143*	*0.143*	*0.143*	*0.143*							
2, 2, 1, 1, 1	0.300	0.300	0.133	0.133	0.133		1.05	1.05	0.93	0.93	0.93	
	0.286	0.286	0.143	0.143	0.143		1.00	1.00	1.00	1.00	1.00	
	0.286	*0.286*	*0.143*	*0.143*	*0.143*							
3, 2, 2, 1, 1	0.400	0.233	0.233	0.067	0.067		1.20	1.05	1.05	0.60	0.60	
	0.385	0.231	0.231	0.077	0.077		1.16	1.04	1.04	0.69	0.69	
	0.333	*0.222*	*0.222*	*0.111*	*0.111*							
2, 1, 1, 1, 1, 1	0.333	0.133	0.133	0.133	0.133	0.133	1.17	0.93	0.93	0.93	0.93	0.93
	0.333	0.133	0.133	0.133	0.133	0.133	1.17	0.93	0.93	0.93	0.93	0.93
	0.286	*0.143*	*0.143*	*0.143*	*0.143*	*0.143*						
4, 1, 1, 1, 1, 1	0.666	0.067	0.067	0.067	0.067	0.067	1.50	0.60	0.60	0.60	0.60	0.60
	0.750	0.050	0.050	0.050	0.050	0.050	1.69	0.45	0.45	0.45	0.45	0.45
	0.444	*0.111*	*0.111*	*0.111*	*0.111*	*0.111*						
3, 2, 1, 1, 1, 1	0.400	0.200	0.100	0.100	0.100	0.100	1.20	0.90	0.90	0.90	0.90	0.90
	0.393	0.179	0.107	0.107	0.107	0.107	1.18	0.81	0.96	0.96	0.96	0.96
	0.333	*0.222*	*0.111*	*0.111*	*0.111*	*0.111*						
2, 2, 2, 1, 1, 1[a]	0.233	0.233	0.233	0.100	0.100	0.100	1.05	1.05	1.05	0.90	0.90	0.90
	0.233	0.233	0.233	0.100	0.100	0.100	1.05	1.05	1.05	0.90	0.90	0.90
	0.222	*0.222*	*0.222*	*0.111*	*0.111*	*0.111*						
3, 3, 2, 1, 1, 1	0.300	0.300	0.200	0.067	0.067	0.067	1.10	1.10	1.10	0.74	0.74	0.74
	0.286	0.286	0.214	0.071	0.071	0.071	1.05	1.05	1.18	0.78	0.78	0.78
	0.273	*0.273*	*0.182*	*0.091*	*0.091*	*0.091*						
4, 2, 2, 1, 1, 1	0.467	0.167	0.167	0.067	0.067	0.067	1.71	0.92	0.92	0.74	0.74	0.74
	0.462	0.154	0.154	0.077	0.077	0.077	1.69	0.85	0.85	0.85	0.85	0.85
	0.364	*0.182*	*0.182*	*0.091*	*0.091*	*0.091*						
3, 2, 2, 2, 1, 1[a]	0.300	0.167	0.167	0.167	0.100	0.100	1.10	0.92	0.92	0.92	1.10	1.10
	0.300	0.167	0.167	0.167	0.100	0.100	1.10	0.92	0.92	0.92	1.10	1.10
	0.273	*0.182*	*0.182*	*0.182*	*0.091*	*0.091*						
4, 3, 3, 1, 1, 1[b]	0.367	0.267	0.267	0.033	0.033	0.033	1.22	1.19	1.19	0.40	0.40	0.40
	0.346	0.269	0.269	0.039	0.039	0.039	1.15	1.20	1.20	0.47	0.47	0.47
	0.300	*0.225*	*0.225*	*0.083*	*0.083*	*0.083*						
3, 3, 2, 2, 2, 1[ab]	0.234	0.234	0.167	0.167	0.167	0.033	1.03	1.03	1.10	1.10	1.10	0.36
	0.234	0.234	0.167	0.167	0.167	0.033	1.03	1.03	1.10	1.10	1.10	0.36
	0.227	*0.227*	*0.152*	*0.152*	*0.152*	*0.091*						

[a] = Non-homogeneous game.
[b] = Corner-solution in legislative bargaining game.

concepts.[13] Choice of voting weights has become increasingly controversial as the European Union has expanded the number of countries and grown in authority. In the most recent round of reform, proposed at the Nice Summit in December 2000, the delegations came armed with computer programs to gauge whether they would be winners or losers under the Nice accord.[14]

[13] See note 3 above.

[14] "Nice Summit: who hopes to join the EU and when, the issues, the objectives, dilemmas and results," *Sunday Times of London,* December 10, 2000, page 1GN. "Every delegation, including Britain's, took special software to Nice to work out majorities, blocking thresholds, and fairness ratios"—the building blocks for power indices. Ian Black, "EU tries to figure out what it decided at Nice," *The Guardian,* December 22, 2000. Viewed on-line through the

TABLE 2—POWER INDICES VERSUS EXPECTED PAYOFFS IN THE EUROPEAN COMMUNITY (EUROPEAN UNION)

Original European Community 1957–1973	France, Germany, Italy	Belgium, Netherlands	Luxembourg		Quota
Voting weights	4	2	1		12/17
Normalized[a]	0.235	0.118	0.059		0.705/1
Minimum integer weights	2	1	0		6/8
Normalized[a]	0.250	0.125	0.000		0.75/1
Banzhaf indices	0.238	0.143	0.000		—
Expected payoffs	0.238	0.119	0.048		—
Expanded European Community 1973–1981	**France, Germany, Italy, UK**	**Belgium, Netherlands**	**Denmark, Ireland**	**Luxembourg**	**Quota**
Voting weights	10	5	3	2	41/58
Normalized[a]	0.172	0.085	0.052	0.034	0.707/1
Minimum integer weights	6	3	2	1	25/35
Normalized[a]	0.171	0.086	0.057	0.029	0.714/1
Banzhaf indices	0.167	0.091	0.066	0.016	—
Expected payoffs	0.170	0.085	0.057	0.035	—

[a] These are also expected payoffs in bargaining games with proportional recognition probabilities.

Table 2 compares the Banzhaf index with expected payoffs in the bargaining game for two periods in the history of the European Community.[15] In the original European Community, the distribution of votes was France, 4, Germany, 4, Italy, 4, Belgium, 2, the Netherlands, 2, and Luxembourg, 1, with 12 of 17 votes required to pass a measure. The corresponding minimum integer weights, which are the relevant weights for the analysis of the game, are (2, 2, 2, 1, 1, 0), with six of eight votes required to pass a measure. As the table shows, the value of the Banzhaf index for Luxembourg is 0 because as a "dummy" player it could never be pivotal in any minimum winning coalition.[16] Belgium and the Netherlands have more power than their vote weights (0.118) or than their minimum integer weights (0.125). By contrast, the competitive bargaining model predicts a nearly proportional pattern of expected payoffs to Council members. The expected payoffs are not strictly proportional because we reach a corner equilibrium (with a corresponding $\theta = 20/21$). Note that although the dummy player receives much more than its Banzhaf value of zero, its expected payoff is slightly less than its vote share in the original game.

Expansion of the European Community in 1973 added the United Kingdom, Denmark, and Ireland. The new system gave 10 votes each to France, Germany, Italy, and the United Kingdom; 5 votes each to Belgium and the Netherlands; 3 votes each to Denmark and Ireland; and 2 votes to Luxembourg. At least 41 of 58 votes were required to pass a measure. Luxembourg is no longer a dummy, but its Banzhaf index of 0.016 is much lower than its vote share of 0.034. By contrast, the competitive bargaining model gives Luxembourg an expected value of 0.035.[17] Note that the competitive bargaining payoffs are nearly linear with respect to the original voting weights. Also, the addition of other countries decreases each original member's expected payoff.

Dow-Jones database, May 2, 2002. Quentin Peel, "Comment & Analysis: Europe's guaranteed gridlock—The absurdly complex voting system of the Nice Treaty," *Financial Times,* July 9, 2001. Viewed on-line through the Dow-Jones database, May 2, 2002.

[15] Brams and Affuso (1985), Lane and Maeland (1995), Felsenthal and Machover (1997, 2000), and others have calculated the power indices for the Council in each of these periods.

[16] Luxembourg also has zero power when recognition probabilities are proportional to the minimum integer weights, since it is then never recognized and never included in a coalition.

[17] The minimum integer weights are (6, 6, 6, 6, 3, 3, 2, 2, 1), and the quota is 25 votes. The equilibrium is at a corner.

These examples illustrate a paradoxical feature of the power indices. Luxembourg's Banzhaf value grew from zero to 0.016, even though its vote share shrank from 1/17 to 2/58. This might reflect the subtle nature of power, or it might reflect problems with the Banzhaf index, such as those noted by Holler (1982, 1987) and Garrett and Tsebelis (2001). Our noncooperative bargaining model with an endogenous agenda apparently does not have this feature. Luxembourg's vote share fell and its power fell. It is ultimately an empirical matter whether this model better captures the essence of collective decision making in the European Union.

III. Evidence from Coalition Governments

An extensive amount of literature uses coalition governments as a proving ground for cooperative and noncooperative bargaining models.[18] In this work, the players are the parties in a parliament. That is, each party's members are assumed to vote as a bloc and therefore each party can be treated as a single player in a weighted voting game. The "dollar" to be divided is the collection of cabinet positions in the government, typically 10 to 30 posts.

In this section, we study the relationship between *voting weights* and *cabinet posts* to test basic predictions of the model analyzed above. We study coalition governments from 1946 to 2001 in Australia, Austria, Belgium, Denmark, Finland, Germany, Iceland, Ireland, Italy, Luxembourg, the Netherlands, Norway, Portugal, and Sweden. This is approximately the same set of countries and governments used in previous studies.[19]

One key prediction of the model above is that the party that is recognized to form a coalition—the *formateur*—will receive a share of the cabinet posts that is much larger than its share of the voting weight. In the previous section, we considered the ex ante share of the value that each player is predicted to receive. Actual coalitions reflect the ex post division, conditional on a specific party being chosen formateur.

The three-party legislatures illustrate the difference between the ex ante and ex post divisions, and lend considerable support for the model. In our sample, 61 parliaments had exactly three non-dummy parties in the parliament and therefore a (1, 1, 1) distribution of voting weights. In all of these cases a minimal winning (i.e., two-party) coalition government formed. On average, the formateur in these cases received 65 percent and the partner received 35 percent of the cabinet posts. The prediction of the Baron-Ferejohn model is 66.7 percent for the formateur and 33.3 percent for the partner. Thus, the formateur took almost exactly the predicted share. The strong formateur effect indicates that the Baron-Ferejohn model of legislative bargaining captures an essential feature of coalition formation.

Previous empirical studies claim to find little or no evidence of a formateur effect. Browne and Franklin (1973) study the relationship between a party's share of cabinet posts and its share of parliamentary seats in the government. They find a nearly linear pattern, a result that some scholars have called "Gamson's Law."[20] The parties that form the governments, they observe, are larger and tend to receive less than an equal share of posts, based on their seat shares. Warwick and Druckman (2001) regress each party's share of posts on its share of seats along with an indicator of whether that party formed the government. They find a negative formateur effect. Some theorist have interpreted these results to imply that there is no bargaining advantage to being chosen to pro-

[18] See Eric Browne and Mark N. Franklin (1973), Browne and John P. Frendreis (1980), Schofield and Michael Laver (1985), and Paul V. Warwick and James N. Druckman (2001). See also Laver and W. Ben Hunt (1992).

[19] For example, Warwick and Druckman (2001) study all of these countries except Australia and Portugal, over the period 1946–1989. Data on party seat shares and government cabinet post allocations are from the following sources: Browne and Dreijmanis (1982), Müller and Strom (2000), and the *European Journal of Political Research* "Political Data Yearbook" special issues, 1992–2001. Following previous researchers, we include all coalition governments, including minority governments and non-minimal-winning governments, except a few cases where one party had an outright majority but still formed a coalition government. These were in the period immediately following World War II in the former axis powers, Austria, Italy, and West Germany, plus a few in Australia.

[20] William A. Gamson (1961) predicted such a relationship based on an informal argument.

TABLE 3—VOTING WEIGHTS AND THE ALLOCATION OF CABINET POSTS IN PARLIAMENTARY GOVERNMENTS, 1946–2001
(*Dep. var. = share of cabinet posts*)

	Unweighted		Weighted	
	(1)	(2)	(1)	(2)
Formateur dummy	0.15*	—	0.24*	—
	(0.046)		(0.041)	
Share of voting weight in Parliament	1.11*	—	1.01*	—
	(0.130)		(0.120)	
Formateur predicted payoff	—	0.66*	—	0.73*
		(0.036)		(0.034)
Partner predicted payoff	—	1.04*	—	0.90*
		(0.200)		(0.179)
Constant	0.07*	0.08*	0.06*	0.08*
	(0.020)	(0.028)	(0.017)	(0.026)
R^2	0.72	0.69	0.81	0.78
# Observations	682	682	682	682

Notes: Clustered standard errors in parentheses, where each cluster is a country. * Statistically significant at the 0.01 level.

pose a government and as evidence against Baron-Ferejohn–type models.[21]

A significant problem with these empirical studies is that they use *seat shares* rather than *voting weights* as the independent variable. From a game-theoretic point of view, voting weights are more relevant for making predictions—virtually all bargaining results are expressed in terms of voting weights, not seat shares. In games with large numbers of players the difference between voting weight shares and seat shares is small, but most parliamentary games involve only a handful of players, and in these cases the differences are important. In our data, for example, 50 percent of the parliaments have six or fewer parties. The correlation between seat shares and voting weights is positive, but not extraordinarily high (0.88). Parties with quite different seat shares often have the same voting weight. In addition, there is a tendency for larger parties to have voting weights smaller than their share of seats.[22]

Using voting weights as the independent variable leads to completely different conclusions about the importance of the formateur. The first column of Table 3 presents a regression of a party's share of cabinet posts on a party's voting weight, plus a dummy variable for whether that party is formateur. This column is similar to the specifications in previous regression analyses, such as those by Browne and Franklin (1973) and Warwick and Druckman (2001), but uses voting weights instead of seat shares. The coefficient on Formateur Dummy is 0.15 with a standard error of 0.046. Thus, on average the party that forms the government receives 15 percent more of the cabinet posts than one would expect from its voting weight alone.[23] Note that in all specifications we cluster the observations by country. This corrects the standard errors for serial dependence of parties within governments and also for serial dependence within countries over time.

[21] For example, Morelli writes: "All the models based on Baron and Ferejohn (1989) yield a disproportionate payoff share for the proposal maker regardless of the distribution of seats, and hence they are not consistent with the basic empirical findings. ... In contrast, the demand bargaining game introduced here performs very well" (Morelli, 1999, p. 810).

[22] Take, for example, the Austrian government of 1986. In the parliament, the Social Democrats held 90 seats; the People's Party, 81 seats; and the Freedom Party, 12 seats. The shares of seats for the Social Democrats, People's Party, and Freedom Party were 0.49, 0.44, and 0.07, respectively. Their shares of the voting weights in the parliament, by contrast, were 0.33, 0.33, and 0.33.

[23] The estimates are nearly the same when we drop minority governments. Note also that if we replicate the existing regression analyses with our data, using seat shares instead of voting weights, then we find similar results to those in Browne and Franklin (1973), Browne and Frendreis (1980), and Warwick and Druckman (2001). The estimated coefficient on the Formateur dummy is small and statistically insignificant (coeff. = 0.009, s.e. = 0.020).

The formateur effect in column 1 provides strong evidence that the parties chosen to form a coalition typically receive more than their voting weight. Even this coefficient probably underestimates the value of being formateur, however, because some cabinet ministries are more valuable than others and the formateur typically controls a disproportionate share of the more valuable posts. Laver and Hunt (1992) find that parties uniformly rank the prime minister as the most valuable post, followed in most cases by finance and foreign affairs. As Warwick and Druckman (2001) point out, the parties that form governments virtually always take the office of prime minister, and they also typically take finance or foreign affairs, or both. Following Warwick and Druckman (2001), we experimented with various ways of assigning higher payoffs to the more important ministries. In all cases, we find that the estimated value of being formateur increases. Column 3 show the results when we assign a value of 3 to prime minister, 2 to finance and foreign affairs, respectively, and 1 to all other ministries. The estimated coefficient on Formateur Dummy increases to 0.24.[24]

We can push the analysis one step farther. Taking the bargaining model literally implies a specific functional form. Suppose the total voting weight in the parliament is w, and the formateur's voting weight is w_f. If the formateur forms a government with a total voting weight of w_g, then it is predicted to receive a payoff of $(w - w_g + w_f)/w$. If a party with voting weight w_i is included as a partner in the coalition, then it is predicted to receive w_i/w. We can, therefore, regress a party's share of cabinet posts on two variables: (a) an indicator of whether the party is formateur times $(w - w_g + w_f)/w$, which we call Formateur Predicted Payoff; and (b) an indicator of whether the party is not formateur times w_i/w, which we call Partner Predicted Payoff. If the bargaining model that we have analyzed is *exactly* right, then a regression of a party's share of cabinet posts on these two variables should have an intercept of 0, and the coefficients on both variables should equal 1.

Column 2 in the table presents the coefficient estimates for this specification. The exact specification clearly fails. The coefficient on Formateur Predicted Payoff is approximately 0.66. Thus, although formateurs get more than their "fair share," they receive somewhat less than predicted by the specific game form that we have analyzed. Interestingly, the coefficient on Partner Predicted Payoff is quite close to its predicted value of 1; however, the intercept is significantly different from 0. Column 4 presents the regression estimates using the 3-2-1 weighting described above. The coefficient on Formateur Predicted Payoff rises to 0.73 and the coefficient on Partner Predicted Payoff falls to 0.90.[25]

There is a variety of reasons not to expect a perfect fit. First, as noted above, we are unsure of the value of different ministerial posts. Sharp tests of specific models, therefore, require additional and accurate information about the value of government positions. Second, there may be restrictions on the sorts of coalitions that can form owing to particular ideological battles. For example, postwar European governments almost never include the Communist Party. In some countries, the "effective" bargaining environment may be a restricted variant of the full game, in which certain parties are excluded. In these case, the players' "effective" voting weights and proposal probabilities are different from those of the full game. Third, the random recognition assumption of the model may be wrong. In some countries, such as Austria, the largest party is always recognized first.[26] Fourth, the model assumes a closed rule (i.e., the formateur makes offers that cannot be amended); an open rule may be more appropri-

[24] It is difficult to measure the actual value of different ministries; this is an important subject for further empirical inquiry. Several previous analyses test the linearity prediction as well, by including squared terms or step-function terms. Without an accurate measure of the value of different posts, however, these tests are probably biased.

[25] We tried a variety of weighting schemes. Giving higher values to Prime Minister, Finance, and Foreign Affairs, we typically find that there is no statistically significant difference between the slopes on Formateur Predicted Payoff and Partner Predicted Payoff, but that both slopes are below 1 and the constant is always positive.

[26] On the other hand, Diermeier and Merlo (2004) provide evidence that the assumption of random recognition with probabilities proportional to seat shares provides a good first approximation to the frequency with which parties act as formateur.

ate. These represent important areas for further empirical and theoretical investigation.[27]

Overall, however, the patterns of coalition formation in European governments provide strong evidence for the two main qualitative predictions of the Baron-Ferejohn approach. The party recognized to form a coalition has added leverage and thus gets more out of a given situation than its vote share. Moreover, the "price" to the formateur of including a party in a coalition is linear in its voting weight.

IV. Discussion

In this paper, we characterize the equilibrium of the Baron-Ferejohn model under weighted voting. The analytical advantage of this noncooperative approach is that it endogenizes the "cost" of coalition partners. As a result, the model predicts that any actor with k votes has the same price or expected value in vote trading as k actors with one vote each. This prediction differs from that of conventional power indices, which have been the main approach to the study of weighted voting.

We also find empirical support for the general approach embodied in this model using data on coalition government formation from 1946 to 2001. The model predicts that the player chosen to form a coalition will receive disproportionately more than his or her voting weight in the ex post distribution of payoffs. We conduct the first statistical analysis of the relationship between *voting weights* and seat shares, and find a large advantage to being formateur. To the extent that the estimated relationships deviate from the model's predictions, small parties receive higher payoffs than predicted, and formateurs appear not to extract the full advantage of their privileged positions. These more detailed findings are suggestive, but tentative due to our inability to measure accurately the value of different cabinet posts. Further empirical research is clearly required.

The model is clearly quite stylized. Several extensions and further generalizations merit special attention. First, our results suggest that proportional payoffs are sustainable by a wide range of proposal probabilities. A more general model is needed, however, to examine the interaction between proposal power and voting power. Second, as a game of pure distribution, it excludes situations in which players' preferences are correlated. For example, players may have spatial or ideological preferences. Any player "close" to the formateur, then, receives additional benefits from the formateur having been chosen. Spatial neighbors, then, may be cheaper partners. Third, the model does not consider a number of common institutional features. The game employs a closed rule, allowing no amendments to a proposal. The open rule introduces analytical complications that have yet to be solved. Also, in most contexts the formation of the agenda is often not random, but is shaped by political parties or legislative committees. We conjecture that any model in which there is a residual claimant, such as the formateur, will have similar predictions. In particular, the price-theoretic substitution argument should hold quite generally.

[27] Of course, another problem is that the model predicts minimal winning coalitions, but we do not always observe them. Also, we assumed an "interior" equilibrium for all cases.

Appendix

PROOF OF PROPOSITION 1:

Existence follows from Theorem 1 of Banks and Duggan (2000). It is sufficient to verify that our game satisfies the Limited Shared Weak Preference (LSWP) property of Banks and Duggan.

For any legislator i and outcome of $\mathbf{x} \in X$, let $R_i(\mathbf{x})$ and $P_i(\mathbf{x})$ denote the weak and strict upper contour sets of $\mathbf{x}$, respectively. For any coalition $\mathbf{C}$, let $R_{\mathbf{C}}(\mathbf{x}) = \cap_{i \in \mathbf{C}} R_i(\mathbf{x})$ and $P_{\mathbf{C}}(\mathbf{x}) = \cap_{i \in \mathbf{C}} P_i(\mathbf{x})$ denote the set of alternatives weakly and strongly preferred to $\mathbf{x}$ by all members of $\mathbf{C}$, respectively. Let $\overline{P_{\mathbf{C}}}(\mathbf{x})$ be the closure of $P_{\mathbf{C}}(\mathbf{x})$. LSWP holds if, for all $\mathbf{C}$ and $\mathbf{x}$, $|R_{\mathbf{C}}(\mathbf{x})| > 1$ implies $R_{\mathbf{C}}(\mathbf{x}) \subseteq \overline{P_{\mathbf{C}}}(\mathbf{x})$.

To show this, note that $|R_{\mathbf{C}}(\mathbf{x})| > 1$ iff $\Sigma_{i \in \mathbf{C}}\, x_i < 1$. Thus, for all such $\mathbf{x}$ and $\mathbf{C}$, $P_{\mathbf{C}}(\mathbf{x}) = \{\mathbf{x}' \in$

$X|x_i' > x_i\ \forall i \in \mathbf{C}\} \neq \emptyset$. Since X is a compact subset of $\Re^n$, the closure of $P_{\mathbf{C}}(\mathbf{x})$ is $\overline{P_{\mathbf{C}}(\mathbf{x})} = \{\mathbf{x}' \in X|x_i' \geq x_i\ \forall i \in \mathbf{C}\}$. Hence $\overline{P_{\mathbf{C}}(\mathbf{x})} = R_{\mathbf{C}}(\mathbf{x})$, which establishes the result.

PROOF OF COMMENT 1:

We begin by defining some useful notation. Let ν_i ($i \in \{1, \ldots, n\}$) represent the continuation value for legislator i. Let ϑ_i be the probability that legislator i is chosen as a coalition partner, given that she is not the proposer. Then, $\nu_i = p_t(1 - \underline{\nu}_i) + (1 - p_t)\vartheta_i\nu_i$, where $\underline{\nu}_i$ is the minimum total payment proposed to coalition partners besides herself. Rearranging, $\nu_i = (1 - \underline{\nu}_i)/[1/p_t - (1/p_t - 1)\vartheta_i]$.

Suppose that legislators j and k are of type t, and $\nu_j > \nu_k$. Also, without loss of generality, let $\nu_k = \min\{\nu_i|t(i) = i\}$. Then $\nu_j > \nu_k$ may be written as $(1 - \underline{\nu}_j)/[1/p_t - (1/p_t - 1)\vartheta_j] > (1 - \underline{\nu}_k)/[1/p_t - (1/p_t - 1)\vartheta_k]$. For this to be true, either: (i) $\vartheta_j > \vartheta_k$, or (ii) $\underline{\nu}_j < \underline{\nu}_k$ (or both). We show that neither of these holds, addressing them in order.

The probability of being chosen as a coalition partner can be represented as the sum of the probabilities of being chosen partner by each of the other players, weighted by the recognition probability. Let legislator i be of type t, and let ϑ_i^m be the probability that i is chosen as a coalition partner, given that legislator $m \neq i$ is the proposer. Then $\vartheta_i = [1/(1 - p_t)] \sum_{m \neq i} p_{t(m)}\vartheta_i^m$. We show that each element of these sums is such that $\vartheta_j^m \leq \vartheta_k^m$. Consider, first, the situations where the proposer is $m \neq j, k$. The condition $\nu_j > \nu_k$ implies that in any coalition formed by m, legislator j will be included only if legislator k is also included, because for the same number of votes the proposer would choose the lower cost partner. Therefore, $\vartheta_j^m \leq \vartheta_k^m$ for any proposer $m \neq j, k$. Second, consider the situations in which j or k is the proposer (these are the remaining probabilities in the calculation of ϑ_j and ϑ_k). It is sufficient to show $\vartheta_j^k \leq \vartheta_k^j$. If $\vartheta_k^j = 1$, then the result is established. So, suppose $\vartheta_k^j < 1$. Then there exists a cheapest set of coalition partners $\mathbf{C}_j$ for proposer j such that $k \notin \mathbf{C}_j$ and $\mathbf{C}_j \cup \{j\} \in \mathcal{W}$. Since proposer k may select $\mathbf{C}_j$ as coalition partners and $\mathbf{C}_j \cup \{k\} \in \mathcal{W}$, she includes legislator j as a coalition partner only if there exists some set of coalition partners $\mathbf{C}_k \ni j$ satisfying $\mathbf{C}_k \cup \{k\} \in \mathcal{W}$ and $\sum_{i \in \mathbf{C}_j} \nu_i \geq \sum_{i \in \mathbf{C}_k} \nu_i$. But then $\nu_j > \nu_k$ implies the existence of a set of coalition partners for j $\mathbf{C}_k' = (\mathbf{C}_k \backslash \{j\}) \cup \{k\}$ satisfying $\sum_{i \in \mathbf{C}_k} \nu_i > \sum_{i \in \mathbf{C}_k'} \nu_i$. Since $\sum_{i \in \mathbf{C}_k'} \nu_i \geq \sum_{i \in \mathbf{C}_j} \nu_i$, we have $\sum_{i \in \mathbf{C}_k} \nu_i > \sum_{i \in \mathbf{C}_j} \nu_i$ and so no such $\mathbf{C}_k$ exists. Thus, legislator j cannot belong to any least cost set of coalition partners for proposer k; i.e., $\vartheta_j^k = 0$, establishing the result. Therefore, $\vartheta_j \leq \vartheta_k$.

Turning to (ii), let $\delta = \nu_j - \nu_k$. If there exists a minimum-cost set of coalition partners $\mathbf{C}_j$ for proposer j such that $k \notin \mathbf{C}_j$ and $\mathbf{C}_j \cup \{j\} \in \mathcal{W}$, then $\mathbf{C}_j$ is available to proposer k, so $\underline{\nu}_k \leq \underline{\nu}_j$. Otherwise, the coalition partners $\mathbf{C}_j \backslash \{k\}$ are available to proposer k and $\underline{\nu}_k \leq \underline{\nu}_j + \nu_j - \nu_k$. Thus, $\underline{\nu}_k - \underline{\nu}_j \leq \delta$, or equivalently $(1 - \underline{\nu}_j) - (1 - \underline{\nu}_k) \leq \delta$. This implies $(1 - \underline{\nu}_j)/z' - (1 - \underline{\nu}_k)/z'' = \delta$ for $z' \geq z'' \geq 1$ only if $z' = z'' = 1$. Using the result of (i), this implies $(1 - \underline{\nu}_j)/[1/p_t - (1/p_t - 1)\vartheta_j] - (1 - \underline{\nu}_k)/[1/p_t - (1/p_t - 1)\vartheta_k] = \delta$ only if $\vartheta_j = \vartheta_k = 1$. Now note that $\vartheta_j = 1$ implies $\nu_j = 1 - \underline{\nu}_j$, or equivalently $\nu_j + \sum_{i \in \mathbf{C}_j} \nu_i = 1$. Since $\underline{w} < w - w_T$, we have $\mathbf{C}_j \cup \{j\} \neq \mathbf{N}$, and thus there exists some legislator l such that $\nu_l = 0$. If l is recognized as proposer, then since $\mathbf{N} \backslash \{j\} \in \mathcal{W}$, we have $\underline{\nu}_l \leq 1 - \nu_j$, and hence $\nu_l \geq (1 - \underline{\nu}_l)/(1/p_{t(l)}) \geq \nu_j/(1/p_{t(l)}) > 0$: contradiction. Thus $\vartheta_j \neq 1$. We conclude that $\underline{\nu}_j < \underline{\nu}_k$ cannot hold, so $\underline{\nu}_j \geq \underline{\nu}_k$.

Therefore, $\nu_j > \nu_k$ implies $\vartheta_j \leq \vartheta_k$ and $\underline{\nu}_j \geq \underline{\nu}_k$, a contradiction.

PROOFS OF PROPOSITIONS 2, 3, AND 4:

To prove Propositions 2, 3, and 4, we employ three lemmas. The lemmas require some additional notation. Recall that the relative price of a type-t legislator is $\theta_t = v_t rw/w_t$. Let $\mathbf{T}_L \equiv \{t \in \mathbf{T}|\theta_t = \min_s\{\theta_s\}, \theta_t \neq 1\}$ denote the (possibly empty) set of "cheapest" types. Where convenient, we will use t' and t'' to denote distinct types, where $\theta_{t'} \leq \theta_{t''}$. Finally, we use $\lceil y \rceil$ and $\lfloor y \rfloor$ to denote the lowest integer greater than or equal to and highest integer lower than or equal to $y \in \Re$, respectively.

LEMMA 1: *In a stationary equilibrium, for any* $\varepsilon > 0$ *there exists a finite* $\bar{r}_\varepsilon$ *such that for any* $t \in \mathbf{T}$ *and* $r \geq \bar{r}_\varepsilon$, $\underline{v}_t \leq (r\underline{w} - w_t)/(rw) + \varepsilon$.

PROOF:

Let $\{t_j\}_{j=1}^T$ represent an ordering of types such that $\theta_{t_a} \leq \theta_{t_b}$ for any $a < b$. For a given r and proposer i of type t, construct a coalition $\mathbf{C}_t \not\ni i$ as follows. Begin with $\mathbf{C}_t = \varnothing$ and add legislators according to the following algorithm:

1. If $w(\mathbf{C}_t) + w_t \geq r\underline{w}$, then $\mathbf{C}_t$ is complete.
2. Otherwise, add a legislator (not i) of type t_m, where $m = \min\{j | n_{t_j}(\mathbf{C}_t \cup \{i\}) < rn_{t_j}\}$, and return to step (1).

By this procedure, $w(\mathbf{C}_t) \geq r\underline{w} - w_t$, and $v(\mathbf{C}_t) \leq w(\mathbf{C}_t)/(rw - w_t)$. To see why the latter statement holds, suppose otherwise. Then $v(\mathbf{N}\backslash\{\mathbf{C}_t \cup \{i\}\}) < w(\mathbf{N}\backslash\{\mathbf{C}_t \cup \{i\}\})/(rw - w_t)$. This implies that there exists a type-t' legislator in $\mathbf{N}\backslash\{\mathbf{C}_t \cup \{i\}\}$ and type-t'' legislator in $\mathbf{C}_t$ such that $\theta_{t'} < \theta_{t''}$, which contradicts the construction of $\mathbf{C}_t$.

Clearly, $v(\mathbf{C}_t) \geq \underline{v}_t$ and, thus, $w(\mathbf{C}_t)/(rw - w_t) \geq \underline{v}_t$. Also, by the algorithm above, $w(\mathbf{C}_t)$ exceeds $r\underline{w} - w_t$ by less than w_T, where w_T is the weight of the most expensive type. So $w(\mathbf{C}_t) < r\underline{w} - w_t + w_T - 1$. Then $\underline{v}_t \leq (r\underline{w} - w_t)/(rw) + \varepsilon$ if

$$\text{(A1)} \qquad \frac{r\underline{w} - w_t + w_T - 1}{rw - w_t} - \frac{r\underline{w} - w_t}{rw} < \varepsilon_1 + \varepsilon_2 = \varepsilon$$

for r sufficiently large and some $\varepsilon_1, \varepsilon_2 > 0$. It is then sufficient to verify that:

$$\text{(A2)} \qquad \frac{r\underline{w} - w_t}{rw - w_t} - \frac{r\underline{w} - w_t}{rw} < \varepsilon_1,$$

$$\text{(A3)} \qquad \frac{w_T - 1}{rw - w_t} < \varepsilon_2.$$

Simplifying, (A2) is satisfied for $r \geq r' = \lceil (w_t/(2w^2\varepsilon_1))(w\varepsilon_1 + \underline{w} + \sqrt{(\underline{w} + w\varepsilon_1)^2 - 4w^2\varepsilon_1}) \rceil$, and (A3) is satisfied for $r \geq r'' = \lceil (w_t/w) + (w_T - 1)/(\varepsilon_2 w) \rceil$. Thus, letting $\bar{r}_\varepsilon = \max\{r', r''\}$ establishes the result.

LEMMA 2: *In a stationary equilibrium, there exists a finite $\bar{r}_c$ such that for any $r \geq \bar{r}_c$, $\sum_{t \in \mathbf{T}_L} w_t n_t \geq \underline{w}$.*

PROOF:

We show that $\sum_{t \in \mathbf{T}_L} w_t n_t < \underline{w}$ for r sufficiently large implies that $q_{t'} \to 1$ for any $t' \in \mathbf{T}_L$, which in turn implies $t' \notin \mathbf{T}_L$, an obvious contradiction.

If $\sum_{t \in \mathbf{T}_L} w_t n_t < \underline{w}$, then a least-cost winning coalition $\mathbf{C}$ must contain at least $\lceil r/w_T \rceil - 1$ legislators of types not in $\mathbf{T}_L$, not including the proposer. $\mathbf{C}$ must then contain at least $\lfloor (\lceil r/w_T \rceil - 1)/(T - 1) \rfloor$ legislators of some type $t'' \notin \mathbf{T}_L$. This implies that for any $k \in \mathbb{Z}_+$,

$$\text{(A4)} \qquad \text{if } r \geq w_T(k + 1)(T - 1) \text{ then } n_{t''}(\mathbf{C}) \geq k.$$

Now consider how many legislators of any type $t' \in \mathbf{T}_L$ can be excluded from $\mathbf{C}$. If $n_{t''}(\mathbf{C}) \geq w_{t'}$ and $n_{t'}(\mathbf{N}\backslash\mathbf{C}) \geq w_{t''}$, then the proposer can replace $w_{t'}$ type-t'' legislators in $\mathbf{C}$ with $w_{t''}$ type-t' legislators in $\mathbf{N}\backslash\mathbf{C}$ and achieve voting weight $w(\mathbf{C})$ at strictly lower cost. This contradicts the assumption that $\mathbf{C}$ is a least-cost winning coalition. By (A4), $n_{t''}(\mathbf{C}) \geq w_{t'}$ when $r \geq r' \equiv w_T(w_T + 1)(T - 1)$. Thus, $r \geq r'$ implies $n_{t'}(\mathbf{N}\backslash\mathbf{C}) < w_{t''}$, and hence $n_{t'}(\mathbf{N}\backslash\mathbf{C}) < w_T$, for all $t' \in \mathbf{T}_L$.

Suppose $r > r'$. Then for any proposal from any proposer, the probability that a legislator of type $t' \in \mathbf{T}_L$ is included in the proposed coalition is at least $1 - w_T/(rn_{t'} - 1)$ if the proposer is of type t', or at least $1 - w_T/(rn_{t'})$ otherwise. Therefore, $q_{t'} \geq 1 - w_T/(rn_{t'} - 1)$ for all $t' \in \mathbf{T}_L$. This implies that for any $\delta \in (0, 1)$, if $r > r_\delta \equiv \max\{r', (1 + w_T/\delta)/n_{t'}\}$, then $q_{t'} \geq 1 - \delta$.

By (2), the type-t' legislator's continuation value is $v_{t'} = [p_{t'}(1 - \underline{v}_{t'})]/[1 - (1 - p_{t'})q_{t'}]$. By Lemma 1, an upper bound on $\underline{v}_{t'}$ is $[(r\underline{w} - w_{t'})/(rw)] + \varepsilon$. Combining results, for any $\varepsilon > 0$ and $\delta > 0$, if $r > \bar{r}_c \equiv \max\{\bar{r}_\varepsilon, r_\delta\}$, then $v_{t'} \geq [p_{t'}(1 - (r\underline{w} - w_{t'})/rw - \varepsilon)]/[1 - (1 - p_{t'})(1 - \delta)]$. This is greater than $w_{t'}/(rw)$ if

$$\frac{p_{t'}(1 - (r\underline{w} - w_{t'})/(rw) - \varepsilon)}{1 - (1 - p_{t'})(1 - \delta)} > \frac{w_{t'}}{rw}$$

$$p_{t'}(rw - r\underline{w} + w_{t'} - rw\varepsilon) > w_{t'}(p_{t'} + \delta(1 - p_{t'}))$$

$$\text{(A5)} \qquad \delta < \frac{p_{t'}r(w - \underline{w} - w\varepsilon)}{w_{t'}(1 - p_{t'})}.$$

Let $\varepsilon < 1 - \underline{w}/w$, let δ satisfy (A5), and choose r_δ and $\bar{r}_\varepsilon$ accordingly. Then, for all $r > \bar{r}_c$, we have $v_{t'} > w_{t'}/w$, contradicting $t' \in \mathbf{T}_L$.

LEMMA 3: *In a stationary equilibrium, if* $r \geq \bar{r}_c$ *as defined in Lemma* 2, *then* $q_t < 2Tw_T/(rn_t)$ *for all* $t \notin \mathbf{T}_L$.

PROOF:

We first derive a relationship between the number of legislators of any type $t'' \notin \mathbf{T}_L$ that can belong to any least-cost winning coalition $\mathbf{C}$, and the number of legislators of some type t' ($t' \in \mathbf{T}_L$) that can belong to $\mathbf{N}\backslash\mathbf{C}$. Suppose $n_{t''}(\mathbf{C})$ type-t'' legislators are in $\mathbf{C}$. By Lemma 2, there exists $\bar{r}_c$ such that for $r > r_c$, $\sum_{t \in \mathbf{T}_L} w_t n_t(\mathbf{N}\backslash\mathbf{C}) \geq n_{t''}(\mathbf{C})w_{t''}$. Thus, for some type $t' \in \arg\max_{t \in \mathbf{T}_L}\{n_t(\mathbf{N}\backslash\mathbf{C})\}$, $n_{t'}(\mathbf{N}\backslash\mathbf{C}) \geq \lfloor n_{t''}(\mathbf{C})w_{t''}/\sum_{t \in \mathbf{T}_L} w_t \rfloor$.

We now derive an upper bound on $n_{t''}(\mathbf{C})$. Note that if $n_{t''}(\mathbf{C}) \geq w_{t'}$ and $n_{t'}(\mathbf{N}\backslash\mathbf{C}) \geq w_{t''}$, then the proposer can exchange $w_{t'}$ type-t'' legislators in $\mathbf{C}$ for $w_{t''}$ type-t' legislators in $\mathbf{N}\backslash\mathbf{C}$, and achieve voting weight $w(\mathbf{C})$ at strictly lower cost. This contradicts the assumption that $\mathbf{C}$ is a least-cost winning coalition. Thus either $n_{t''}(\mathbf{C}) < w_{t'}$, or $w_{t''} > \lfloor n_{t''}(\mathbf{C})w_{t''}/\sum_{t \in \mathbf{T}_L} w_t \rfloor$, which simplifies to $1 > \lfloor n_{t''}(\mathbf{C})/\sum_{t \in \mathbf{T}_L} w_t \rfloor$, or $n_{t''}(\mathbf{C}) < 2\sum_{t \in \mathbf{T}_L} w_t$. Thus, $n_{t''}(\mathbf{C}) < \max\{w_{t'}, 2\sum_{t \in \mathbf{T}_L} w_t\} < 2Tw_T$ in any least-cost winning coalition.

Aggregating over all proposers and least-cost winning coalitions, this implies $q_{t''} < 2Tw_T/(rn_{t''})$.

PROOF OF PROPOSITION 2:

Suppose that $v_t < \theta_t$ for all types $t \in \mathbf{T}_L$. Note that this implies $\mathbf{T}_L \neq \mathbf{T}$, for otherwise $\sum_t n_t v_t < 1$, an obvious contradiction. We show that for r sufficiently large, no such $\mathbf{T}_L$ exists.

We begin by deriving a lower bound on v_t, in two parts. First, Lemma 1 provides a lower bound on $1 - \underline{v}_t$: for any $\varepsilon > 0$ there exists $\bar{r}_\varepsilon$ such that if $r \geq \bar{r}_\varepsilon$, then $1 - \underline{v}_t > (rw - r\underline{w} + w_t)/(rw) - \varepsilon$.

Second, we derive a lower bound on $q_{t'}$ for some $t' \in \mathbf{T}_L$. Every proposer must assemble a coalition $\mathbf{C}'$ with a weight (not including herself) of at least $r\underline{w} - w_T$. Thus,

$$\sum_{t \in \mathbf{T}_L} rq_t n_t(\mathbf{C}')w_t + \sum_{t \notin \mathbf{T}_L} rq_t n_t(\mathbf{C}')w_t \geq r\underline{w} - w_T.$$

Simplifying and applying Lemma 3 yields:

$$\sum_{t \in \mathbf{T}_L} q_t n_t(\mathbf{C}')w_t > \underline{w} - \frac{w_T}{r} - \frac{2Tw_T}{r}\sum_{t \notin \mathbf{T}_L} n_t(\mathbf{C}')w_t$$

for $r \geq \bar{r}_c$. Thus, for any $\delta > 0$ and for all $r > r_\delta \equiv w_T(1 - 2T\sum_{t \notin \mathbf{T}_L} n_t(\mathbf{C}')w_t)/\delta$, we have $\sum_{t \in \mathbf{T}_L} q_t n_t(\mathbf{C}')w_t > \underline{w} - \delta$. Thus, for $r > \max\{\bar{r}_c, r_\delta\}$ there exists some $t' \in \mathbf{T}_L$ such that $q_{t'} > (\underline{w} -$

$\delta)/\sum_{t\in\mathbf{T}_L} n_t(\mathbf{C}')w_t$. Since $\sum_{t\in\mathbf{T}_L} n_t(\mathbf{C}')w_t < w$, this implies that for δ sufficiently small and $r > \max\{\bar{r}_c, r_\delta\}$, $q_{t'} > \tilde{w}/w$ for some $\tilde{w} > \underline{w}$.

Substituting into (2), $v_{t'} < w_{t'}/(rw)$ implies:

$$\frac{p_{t'}[(rw - r\underline{w} + w_{t'})/(rw) - \varepsilon]}{1 - (1 - p_{t'})\tilde{w}/w} < \frac{w_{t'}}{rw}.$$

Letting $p_{t'} = w_{t'}/(rw)$ and simplifying yields:

$$\frac{w_{t'} - r\underline{w}}{rw} - \varepsilon < -\frac{(1 - p_{t'})\tilde{w}}{w}$$

$$\text{(A6)} \qquad \frac{w_{t'}}{r}\left(\frac{\tilde{w}}{w} - 1\right) > \tilde{w} - \underline{w} - w\varepsilon.$$

The left-hand side of (A6) is negative, while for ε sufficiently small the right-hand side is positive, and thus (A6) cannot hold. Thus, for suitably chosen ε, δ, and $r > \bar{r}_1 \equiv \max\{\bar{r}_c, \bar{r}_\varepsilon, r_\delta\}$, we have $v_{t'} \not< w_{t'}/w$, a contradiction.

PROOF OF PROPOSITION 3:

Since $\sum_t rn_t v_t = 1$, $v_t = w_t/(rw)$ for all t if $\theta_t = 1$ for all t. It is therefore sufficient to provide conditions under which there is no type such that $\theta_t > 1$.

Note that $\theta_t \le 1$ for all $t \in \mathbf{T}_L$, and therefore we check only $t \notin \mathbf{T}_L$. By Lemma 3, $q_t \le 2Tw_T/(rn_t)$ for any $t \notin \mathbf{T}_L$ and all $r > \bar{r}_c$. By Lemma 2 and the definition of $\mathbf{T}_L$, $\underline{v}_t \ge \theta(r\underline{w} - w_t)/(rw)$, where $\theta = \min\{\theta_t\}$. Thus, using (5), for $r > \bar{r}_c$ and $t \notin \mathbf{T}_L$, we obtain the following upper bound on v_t:

$$v_t = \frac{\theta_t w_t}{rw} \le \frac{1 - \theta_t(r\underline{w} - w_t)/(rw)}{rn - (rn - 1)2Tw_T/(rn_t)}.$$

This implies the following upper bound on θ_t:

$$\theta_t \le \frac{w}{\underline{w} + w_t n - w_t/r - (n - 1/r)2Tw_T w_t/(rn_t)}.$$

Thus, $\theta_t > 1$ for $t \notin \mathbf{T}_L$ only if

$$\text{(A7)} \qquad w > \underline{w} + w_t n - \frac{1}{r}\left[w_t + \frac{2Tw_T w_t}{n_t}\left(n - \frac{1}{r}\right)\right].$$

Simple manipulation reveals that $[w_t + (2Tw_T w_t/n_t)(n - 1/r)]/r < \delta$ for type t and any $\delta > 0$ if: $r > r_t(\delta) \equiv \{[2nTw_T + n_t + \sqrt{(2nTw_T + n_t)^2 - 8\delta n_t Tw_T/w_t}]/(2\delta n_t)\}w_t$. Thus, there exists $\eta_t > 0$ such that if $r > r_t(\eta_t)$ then (A7) is satisfied only by $n \le (w - \underline{w})/w_t$.

This implies that for any $r > \bar{r}_2 \equiv \max\{\bar{r}_c, \max_t\{r_t(\eta_t)\}\}$, (A7) holds for type t only if $n \le (w - \underline{w})/w_t$. Since $w_1 = \min_t\{w_t\}$, (A7) cannot hold for any type t if

$$\text{(A8)} \qquad n > \frac{w - \underline{w}}{w_1}.$$

Thus, when $r > \bar{r}_2$ and (A8) holds, there does not exist any type such that $\theta_t > 1$, and hence $\theta_t = 1$ for all t.

PROOF OF PROPOSITION 4:

We first establish sufficient and necessary conditions for an equilibrium in which $\theta_t > 1$ and hence $\mathbf{T}_L \neq \mathbf{T}$, and then show uniqueness of the cutoff type t_0. Let $\theta = \min\{\theta_t\}$. Also, assume throughout that $r > \bar{r}_c$.

We begin by deriving a lower bound on v_t, and use it to derive a sufficient condition for $t \notin \mathbf{T}_L$. This is done by substituting an upper bound on $\underline{v}_t$ and letting $q_t = 0$ in (5). To derive an upper bound on $\underline{v}_t$, note that a type-t proposer must build a coalition with partners $\mathbf{C}$ such that $w(\mathbf{C}) \geq r\underline{w} - w_t$. By Lemma 2, there exists a set of coalition partners $\mathbf{C}'$ such that $w(\mathbf{C}') > r\underline{w} - w_t - w_T$, and $\mathbf{C}' \subset \{i | t(i) \in \mathbf{T}_L\}$. The proposer can then complete the coalition by adding a legislator of type T. Comment 1 implies that $v_T \leq 1/(rn_T)$, and thus $\underline{v}_t < \theta(r\underline{w} - w_t)/(rw) + 1/(rn_T)$.

Substituting into (5) yields the following lower bound on v_t:

$$v_t = \frac{\theta_t w_t}{rw} \geq \frac{1 - \theta_t(r\underline{w} - w_t)/(rw) - 1/(rn_T)}{rn}. \tag{A9}$$

This implies the following lower bound on θ_t:

$$\theta_t \geq \frac{w - w/(rn_T)}{\underline{w} + w_t n - w_t/r}.$$

Thus, $\theta_t > 1$ if

$$w > \underline{w} + w_t n + \frac{1}{r}\left(\frac{w}{n_T} - w_t\right) \tag{A10}$$

$$n < \frac{w - \underline{w} - (w/n_T - w_t)/r}{w_t}. \tag{A11}$$

Note that if $(w - \underline{w})/w_t$ is integral for some type t, then (A11) is satisfied by $n < (w - \underline{w})/w_t$ if $(w/n_T - w_t)/r < w_t$, or $r > w/(n_T w_t) - 1$. Otherwise, if $(w - \underline{w})/w_t$ is not integral for some t, then let $\delta_t = (w - \underline{w})/w_t - \lfloor(w - \underline{w})/w_t\rfloor$. Now (A11) is satisfied by $n < (w - \underline{w})/w_t$ if $(w/n_T - w_t)/r < \delta_t$, or $r > (w/n_T - w_t)/\delta_t$.

We therefore conclude that if $r > r' \equiv \max\{w/(n_T w_1) - 1, \max_t\{(w/n_T - w_t)/\delta_t\}\}$, then

$$n < \frac{w - \underline{w}}{w_t} \tag{A12}$$

is sufficient to ensure that for any t, $\theta_t > 1$ and thus, $t \notin \mathbf{T}_L$ and $\mathbf{T}_L \neq \mathbf{T}$. Since $w_1 = \min_t\{w_t\}$, (A12) will hold for some t if $n < (w - \underline{w})/w_1$. Note that if (A12) holds for some type t', then it must also hold for any $t < t'$.

Next, consider necessary conditions for $t \notin \mathbf{T}_L$. By the proof of Proposition 3, if $r > \bar{r}_2$ then $\theta_t > 1$ for $t \notin \mathbf{T}_L$ only if

$$n \leq \frac{w - \underline{w}}{w_t}. \tag{A13}$$

Putting the results together, suppose that $r > \bar{r}_3 \equiv \max\{\bar{r}_2, r', \bar{r}_c\}$, and let $t_0 = \max\{t | \text{(A13) holds}\}$. Clearly t_0 is unique. Then in a stationary equilibrium, by (A13) $\theta_t = \theta < 1$ ($t \in \mathbf{T}_L$) for $t > t_0$, and by (A12) $\theta_t > 1$ ($t \notin \mathbf{T}_L$) for $t < t_0$. If additionally t_0 satisfies (A12), then $\theta_{t_0} > 1$.

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